

Consequences of the partial restoration of chiral symmetry in AdS/QCD

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Abstract

Chiral symmetry is an essential concept in understanding QCD at low energy. We treat the chiral condensate, which measures the spontaneous breaking of chiral symmetry, as a free parameter to investigate the effect of partially restored chiral symmetry on the physical quantities in the frame work of an AdS/QCD model. We observe an interesting scaling behavior among the nucleon mass, pion decay constant and chiral condensate. We propose a phenomenological way to introduce the temperature dependence of a physical quantity in the AdS/QCD model with the thermal AdS metric.

1 Introduction

Based on the AdS/CFT [1], many successful attempts have been made to construct a holographic model of QCD, in both the bottom-up [2, 3] and the top-down [4] approaches. Baryons are also introduced into both of the approaches [5].

Since it has been generally expected that the physical properties of hadrons undergo a significant change in the finite temperature and/or density environment, finite temperature extension of the approaches have been of great interest. One of the interesting approaches is to use the AdS black hole. However, according to the Hawking-Page transition analysis done in [6], the AdS black hole is unstable at low temperature and the thermal AdS metric is energetically favored in the confining phase [6]. In general, the thermal AdS background will not render any temperature dependence of any physical quantities in confined phase, since the metric, thermal AdS, is not globally modified compared to those at zero temperature, AdS. It is shown in [7] that in low-temperature confined phase, the properties of hadrons show no significant changes compared to zero temperature. This means that, in the light of the Hawking-Page transition, the AdS/QCD model may not be of much use, when it comes to the temperature dependence of physical quantities such as meson and baryon masses. As long as we are taking $N_c \rightarrow \infty$ limit, the observation of the Hawking-Page transition analysis is consistent with large N_c QCD [8]. In real world, however, the properties of hadrons, in confined phase, are modified at finite temperature, see [9, 10, 11, 12, 13] for examples. In the light of the Hawking-Page, we could think of such a temperature dependence as a consequence of large N_c corrections in an AdS/QCD approach. We note here that up to now, however, such large N_c corrections have not been successfully included in AdS/QCD [14]. In this work we propose a simple way to introduce a temperature dependence through the chiral condensate in the framework of the hard wall model [2, 3].

The chiral symmetry of QCD has been playing an important role in hadron physics. A pertinent order parameter for the symmetry is the chiral condensate $\langle \bar{q}q \rangle$. There has been any amount of research on the chiral condensate at finite temperature and density in the framework of QCD effective theories or models. One of the interesting questions regarding the chiral condensate is: what is the effect of the partial restoration of the chiral symmetry on the physical quantities such as meson and baryon masses?

In a bottom-up AdS/QCD model, the chiral condensate is encoded in a 5D profile of a scalar field X that couples to quark bilinear $\bar{q}q$ at the boundary of AdS_5 . The background geometry of the hard wall model [2, 3] is defined as a slice of anti-de Sitter (AdS) metric,

$$ds^2 = \frac{1}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2), \quad z_0 \leq z \leq z_m, \quad (1)$$

where $z_0 \rightarrow 0$. Here z_0 is the UV-cutoff and z_m for the IR-cutoff. In the hard wall mode [2, 3] the vacuum expectation of the scalar $X_0 = \langle X(x, z) \rangle$ is given by $X_0 = c_1 z + c_2 z^3$, where c_1 and c_2 are integration constants to be fixed by boundary conditions. According to an AdS/CFT dictionary, c_1 is identified with current quark mass, $c_1 \sim m_q$, and c_2 is interpreted as the chiral condensate, $c_2 \sim \langle \bar{q}q \rangle$. In this work we will set $c_1 = 0$,

no explicit chiral symmetry breaking, and take c_2 as a free parameter of the model. In the hard wall model, the correspondence $c_2 \sim \langle \bar{q}q \rangle$ is realized by imposing the IR boundary condition: $X_0(z_m)/z_m^3 \sim \langle \bar{q}q \rangle$ [3]. We vary the value of c_2 from a finite value to zero to mimic a chiral symmetry restoration in a specific environment such as QCD at finite temperature.

Our primary goal in this study is to observe how physical quantities depend on the change of the chiral condensate, which is, in turn, supposed to address the consequences of the (partial) restoration of chiral symmetry to the physical quantities of QCD effective theories at low energy. One of the interesting observations as a consequence of chiral symmetry restoration is the scaling behavior of the hadron properties at finite temperature and/or density. For examples, NJL model gives [15, 16]

$$\frac{m_N^*}{m_N} \simeq \frac{m_\sigma^*}{m_\sigma} \simeq \frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle}, \quad (2)$$

and BR scaling [16, 17] reads

$$\frac{m_N^*}{m_N} \simeq \frac{m_\sigma^*}{m_\sigma} \simeq \frac{m_\rho^*}{m_\rho} \simeq \frac{m_\omega^*}{m_\omega} \simeq \frac{f_\pi^*}{f_\pi}. \quad (3)$$

Here \star is for temperature/density dependent quantities.

As described in [2, 3], the chiral condensate in the hard wall model is an integration constant to be fixed by the IR boundary condition: $X_0(z_m)/z_m^3 = c_2 \sim \langle \bar{q}q \rangle$ with $m_q = 0$. It has been known [9, 10, 11, 12, 13] that the value of the chiral condensate changes with temperature. Therefore we can define a hard wall model at finite temperature with the following IR boundary condition: $X_0(z_m)/z_m^3 = c_2 \sim \langle \bar{q}q \rangle^*$. Here $\langle \bar{q}q \rangle^*$ denotes the temperature dependent chiral condensate. This is a proposal in this work as a simple way to introduce a temperature dependence in the hard wall model by imposing a IR boundary condition defined at finite temperature. We cannot, however, determine the temperature dependence of the chiral condensate in a self-consistent way within the hard wall model. Therefore, as done in zero temperature case [2, 3], we have to consider $\langle \bar{q}q \rangle^*$ as an input, and take the temperature dependence of the chiral condensate from a QCD effective theory or lattice QCD study. This is a limitation of the present study. Once the temperature dependence of the chiral condensate is given, however, we can easily obtain the temperature dependence of the other physical quantities such as the nucleon mass and the pion decay constant.

2 Scalings of physical quantities in the hard wall model

The action of the model developed in [2, 3] is, adopting the convention in [2],

$$\begin{aligned} S_5 &= \int d^4x \int dz \mathcal{L}_5 \\ &= \int d^4x \int dz \sqrt{g} \text{Tr} \left[-\frac{1}{4g_5^2} (F_L^2 + F_R^2) \right. \\ &\quad \left. + |DX|^2 + 3|X|^2 \right] \end{aligned} \quad (4)$$

where $D_\mu X = \partial_\mu X - iA_{L\mu}X + iXA_{R\mu}$ and $A_{L,R} = A_{L,R}^a t^a$ with $\text{Tr}(t^a t^b) = \frac{1}{2}\delta^{ab}$. The bulk scalar field is defined by $X = X_0 e^{2i\pi^a t^a}$, where $X_0 \equiv \langle X \rangle$. The background metric of the model is a slice of AdS metric,

$$ds^2 = \frac{1}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2), \quad z_0 \leq z \leq z_m, \quad (5)$$

where $z_0 \rightarrow 0$. Here g_5 is the 5D gauge coupling, $g_5^2 = 12\pi^2/N_c$, z_0 is the UV-cutoff and z_m is for the IR-cutoff. In [2, 3], the IR-cutoff z_m is fixed by the ρ -meson mass: $1/z_m \simeq 320$ MeV. The vector- and axial-vector mesons are defined by

$$\begin{aligned} V_\mu &= \frac{1}{2}(A_L + A_R) \\ A_\mu &= \frac{1}{2}(A_L - A_R). \end{aligned} \quad (6)$$

To define our scaling factor σ , we solve the equation of motion for X_0 ,

$$\left[\partial_z^2 - \frac{3}{z} \partial_z + \frac{3}{z^2} \right] X_0 = 0, \quad X_0 = c_1 z + c_2 z^3, \quad (7)$$

where c_1 and c_2 are integration constants. We define $v(z) \equiv 2X_0$. An AdS/CFT dictionary dictates that c_1 is nothing but the source term, current quark mass m_q , and c_2 should be interpreted as the chiral condensate, the order parameter of chiral symmetry breaking/restoration. In this work, we take $m_q = 0$. Then, we have

$$c_1 = 0, \quad c_2 = \frac{1}{2}\sigma,$$

where σ is a free parameter of the model. Note that $\sigma = \sigma_0 \approx (0.33 \text{ GeV})^3$ is compatible with phenomenology [2]. We scale down σ from σ_0 to zero.

We first consider the scaling of vector and axial-vector meson masses as a function of σ . The relevant equations of motions for the transverse component of the vector and axial-vector bulk fields are, after the Kaluza-Klein reduction of the bulk field, $V_\mu(x, z) = \Sigma_n f_n^V(z) V_\mu^{(n)}(x)$,

$$\left[\partial_z^2 - \frac{1}{z} \partial_z + m_n^2 \right] f_n^V(z) = 0, \quad (8)$$

$$\left[\partial_z^2 - \frac{1}{z} \partial_z + m_n^2 - g_5^2 \frac{v^2}{z^2} \right] f_n^A(z) = 0, \quad (9)$$

where $v(z) = \sigma z^3$. As in [3], we impose the following boundary conditions: $f_n^{V,A}(z_0) = 0$, $\partial_z f_n^{V,A}(z_m) = 0$. Since the equation of motion for the vector is blind to $v(z)$, due to $D_\mu v = \partial_\mu v - 2ivA_\mu$, the 4D mass of the vector meson m_n such as ρ -meson mass will not scale with σ . While, the mass of axial-vector mesons will change with varying σ . We plot the mass of the lowest lying vector and axial vector mesons, ρ and a_1 , in Fig. 1. As in Fig. 1, we have $m_{a_1} \approx m_\rho$ at $R_\sigma^{1/3} (\equiv \sigma^{1/3}/\sigma_0^{1/3}) \approx 0.4$. This means that the role of the term with v^2 in Eq. (9) becomes negligible when $R_\sigma^{1/3} \leq 0.4$.

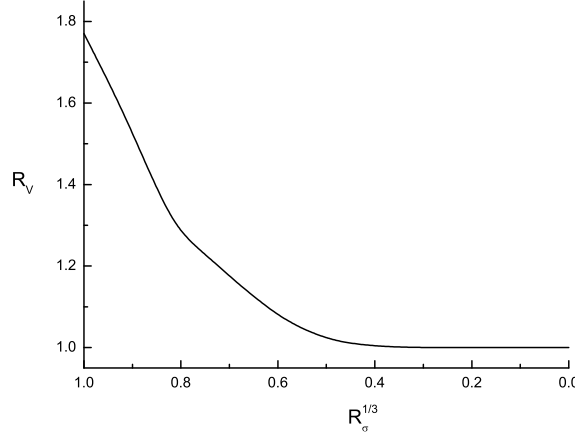


Figure 1: The scaling of a_1 mass normalized to m_ρ as a function of σ . Here $R_V \equiv m_{a_1}(\sigma)/m_\rho$ and $R_\sigma^{1/3} = (\sigma/\sigma_0)^{1/3}$.

Now we discuss the pion decay constant. The pion decay constant is defined by [2]

$$f_\pi^2 = -\frac{1}{g_5^2} \frac{\partial_z A(0, z)}{z} \Big|_{z=z_0}, \quad (10)$$

where $A(0, z)$ is the solution of the following equation,

$$\left[\partial_z^2 - \frac{1}{z} \partial_z - 2 \frac{v^2}{z^2} \right] A(0, z) = 0. \quad (11)$$

The scaling behavior of f_π is shown in Fig. 2 together with that of nucleon.

Finally, we delve into the scaling behavior of the nucleon mass with σ . The model [2, 3] is extended to include baryons in Ref. [18]. The AdS/QCD model of spin $\frac{1}{2}$, isospin $\frac{1}{2}$ baryons is given by the action, referring to [18] for details,

$$\begin{aligned} S_{\text{kin}} &= \int dz \int dx^4 \sqrt{G_5} \left[i \bar{N}_1 \Gamma^M D_M N_1 + i \bar{N}_2 \Gamma^M D_M N_2 - \frac{5}{2} \bar{N}_1 N_1 + \frac{5}{2} \bar{N}_2 N_2 \right], \\ S_m &= \int dz \int dx^4 \sqrt{G_5} \left[-g \bar{N}_1 X N_2 - g \bar{N}_2 X^\dagger N_1 \right], \end{aligned} \quad (12)$$

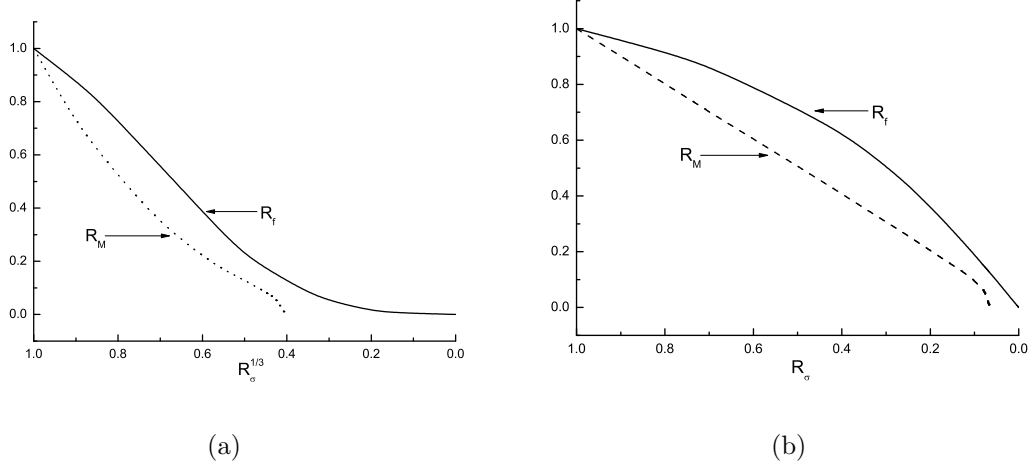


Figure 2: The scaling of the nucleon mass and the pion decay constant as a function of (a) $R_\sigma^{1/3}$, (b) $R_\sigma \equiv \sigma/\sigma_0$. Here $R_M \equiv m_N(\sigma)/m_N(\sigma_0)$ and $R_f \equiv f_\pi(\sigma)/f_\pi(\sigma_0)$.

where the covariant derivatives for N_1 and N_2 include the gauge group $SU(2)_L \times SU(2)_R$ as well as the metric connection, and a single parameter g should be fixed to reproduce the nucleon mass. By expanding N_1 and N_2 in terms of KK modes, it is easy to find the mode equations that must be solved to find the mass spectrum of 4D spin $\frac{1}{2}$ baryons. Writing $N_1(x, z) = f_{1L}(z)B_L(x) + f_{1R}(z)B_R(x)$ and similarly for $N_2(x, z) = f_{2L}(z)B_L(x) + f_{2R}(z)B_R(x)$, where $B_{L,R}$ are the components of the 4D spinor $B = (B_L, B_R)^T$ with mass m_N to be determined, we have

$$\begin{pmatrix} \partial_z - \frac{\Delta}{z} & -\frac{gX_0}{z} \\ -\frac{gX_0}{z} & \partial_z - \frac{4-\Delta}{z} \end{pmatrix} \begin{pmatrix} f_{1L} \\ f_{2L} \end{pmatrix} = -m_N \begin{pmatrix} f_{1R} \\ f_{2R} \end{pmatrix},$$

$$\begin{pmatrix} \partial_z - \frac{4-\Delta}{z} & \frac{gX_0}{z} \\ \frac{gX_0}{z} & \partial_z - \frac{\Delta}{z} \end{pmatrix} \begin{pmatrix} f_{1R} \\ f_{2R} \end{pmatrix} = m_N \begin{pmatrix} f_{1L} \\ f_{2L} \end{pmatrix} \quad (13)$$

with the IR boundary condition $f_{1R}(z_m) = f_{2L}(z_m) = 0$ [18]. Here $\Delta = 9/2$, and $X_0 = \frac{1}{2}\sigma z^3$. The scaling behavior of m_N is shown in Fig. 2. Note that as in the case of axial vector meson mass, a_1 , in Fig. 1, the role of X_0 in the baryon mode equation becomes negligible at $R_\sigma^{1/3} \approx 0.4$ and the mass of nucleon is almost zero at and after $R_\sigma^{1/3} \approx 0.4$. We recast Fig. 2(a) in Fig. 2(b), where $R_\sigma \equiv \sigma/\sigma_0$ is used for the horizontal axis. From Fig. 2(b), we can see the overall scaling behavior of $\frac{f_\pi(\sigma)}{f_\pi(\sigma_0)}$ is not much different from $\frac{\sigma}{\sigma_0}$. A salient feature of the hard wall model is that the vector meson mass, for example ρ -meson mass, is independent of the chiral condensate as shown in Eq. (8), and therefore the mass of ρ is blind to the restoration of chiral symmetry in the present approach. Finally, we comment on the σ -dependence of the scalar meson mass. In general, studying scalar excitations is not simple as they are sensitive to the potential of X . If we follow Ref.[19], where a potential for X is added on the IR, the mass of the first scalar resonance is degenerate with the massless pion as $\sigma \rightarrow 0$.

3 Finite temperature as a boundary condition

In the previous section, we obtain an interesting scaling properties of physical quantities with respect to the chiral condensate in the hard wall model. The chiral condensate is identified as the integration constant, c_2 , fixed by a boundary condition in Eq.(7).

In this section, we propose a phenomenological way to introduce the temperature dependence of physical quantities in the hard wall model by imposing a boundary condition as a function of temperature T . Then we have $c_2 \sim \langle \bar{q}q \rangle^*$, where \star is for temperature dependence. In general, in bottom-up AdS/QCD approach, however, there is no established way to calculate the temperature dependence of σ . Therefore we have to take σ^* as an input and borrow the temperature dependence of σ from a model calculation or lattice QCD study. Although it might be a limitation of the present study, once σ^* is given, we can easily obtain the temperature dependence of the other hadronic parameters such as the nucleon mass and the pion decay constant. Here we focus on the temperature dependence of the pion decay constant. As an example, we take σ^*/σ_0 by extrapolating the

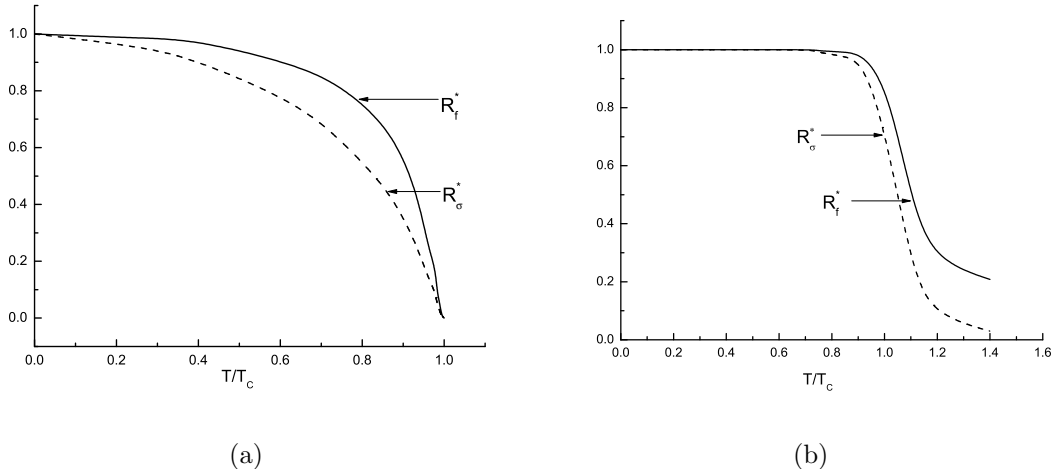


Figure 3: The temperature dependence of the pion decay constant. Here the temperature dependent chiral condensates are inputs, $R_\sigma^* \equiv \sigma^*/\sigma$ and $R_f^* \equiv f_\pi^*/f_\pi$: (a) R_σ^* taken from [9] in the chiral limit, (b) R_σ^* from [10] with a finite current quark mass.

temperature dependence of the chiral condensate obtained from the chiral Lagrangian [9] with two quark flavors in the chiral limit, which is presumably valid at low temperature. Then we can obtain the temperature dependence of f_π using Eqs. (10) and (11) as shown in Fig. 3(a).

In Fig. 3(b), the temperature dependence of the pion decay constant is calculated by adopting the temperature dependence of the chiral condensate obtained in a generalized NJL model [10] with a finite current quark mass.

Now, we compare the temperature dependence of f_π calculated in this model with the one from chiral perturbation theory [11] and from linear and non-linear sigma models

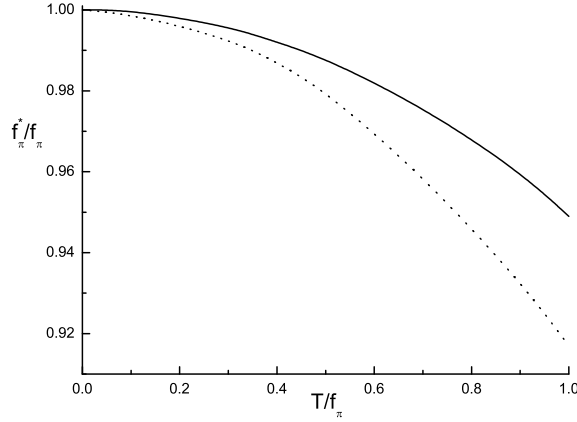


Figure 4: The temperature dependence of the pion decay constant. The temperature dependent chiral condensate is taken from [11, 12], Eq. (14). Solid line is for our result, and dashed line is from [11, 12], given in Eq. (15).

[12]. We note here that at low temperature or at the leading order in temperature T , both studies [11, 12] give the same results on the chiral condensate,

$$\frac{\sigma^*}{\sigma_0} = 1 - \frac{T^2}{8f_\pi^2}. \quad (14)$$

We take the temperature dependent chiral condensate σ^*/σ_0 given in Eq. (14) as an input and calculate the temperature dependence of the pion decay constant using Eqs. (10) and (11). In Fig. 4, we compare our f_π^* with that from [11, 12]. The temperature dependent pion decay constant obtained in [11, 12] at low temperature is given by

$$\frac{f_\pi^*}{f_\pi} = 1 - \frac{T^2}{12f_\pi^2}. \quad (15)$$

The observed temperature dependence of f_π in our work is weaker than the one from [11, 12]. The temperature dependence obtained in the present work is only due to the temperature dependent chiral condensate. In addition to this, we expect some temperature dependence due to large N_c corrections and from higher dimensional terms such as $F_L X F_R X^\dagger$ in the action of the hard wall model.

Finally, we remark that the temperature dependence obtained in the present study has some similarity with that from QCD sum rule at finite temperature, for instance, see [20]. In thermal QCD sum rule, the temperature dependence of condensates, *e.g.*, chiral condensate, is taken from a model study or from lattice QCD, and it is conveyed to the (part of) temperature dependence of physical quantities such as hadron masses. Since the temperature dependent chiral condensate used in this work and the one adopted in the thermal QCD sum rule studies are different, we don't make a direct comparison of

our result with that from [20]. We note here that in thermal QCD sum rule [20], the temperature dependence could have additional sources other than the chiral condensate such as the temperature dependent Wilson coefficients or four-quark condensate.

4 Summary

We have used the hard wall model to study the scaling property of physical quantities as we scale down the chiral condensate. By varying the value of the chiral condensate σ , we study how the other physical parameters scale with σ , which is summarized in Fig. 2. We note here that the mass of ρ -meson is independent of the chiral condensate in the hard wall model [2, 3], and therefore it is blind to the restoration of the chiral symmetry in the present work.

We have introduced the temperature into the AdS/QCD model through the chiral condensate as an IR boundary condition. With a given temperature dependent chiral condensate σ^* , we can easily predict the temperature dependence of other physical quantities such as hadron masses or decay constants. As an example, we have calculated the temperature dependence of the pion decay constant and compare our result with that from chiral perturbation theory at low temperature. We find that the temperature dependence of the pion decay constant predicted from the present study is weak compared to that from chiral perturbation theory [11] and from linear and non-linear sigma models [12].

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